Roots of Real Numbers

What are roots?

$5 \cdot 5 = 25$ so 5 is the square root of 25

$5 \cdot 5 \cdot 5 = 125$ so 5 is the cube root of 125

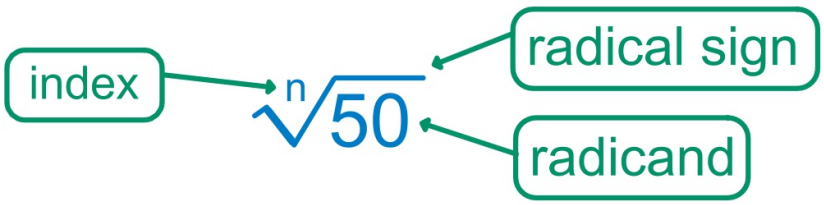
$5 \cdot 5 \cdot 5 \cdot 5 = 625$ so 5 is the fourth root of 625

$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3125$ so 5 is the fifth root of 3125

and so on and so forth....

*n*th root is $a^n = b$

a times itself an *n*th number of times is equal to *b*



Some numbers have more than one real *n*th root. For example, 36 has two square roots, 6 and −6. When there is more than one real root, the nonnegative root is called the **principal root**. When no index is given, as in $\sqrt{36}$, the radical sign indicates the principal square root. The symbol $\sqrt[n]{b}$ stands for the principal *n*th root of *b*. If *n* is odd and *b* is negative, there will be no nonnegative root. In this case, the principal root is negative.

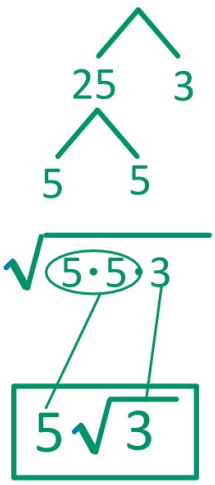
$\sqrt{16} = 4$	$\sqrt{16}$ indicates the principal square root of 16.
$-\sqrt{16} = -4$	$-\sqrt{16}$ indicates the opposite of the principal square root of 16.
$\pm\sqrt{16} = \pm 4$	$\pm\sqrt{16}$ indicates both square roots of 16. ± means positive or negative.
$\sqrt[3]{-125} = -5$	$\sqrt[3]{-125}$ indicates the principal cube root of −125.
$-\sqrt[4]{81} = -3$	$-\sqrt[4]{81}$ indicates the opposite of the principal fourth root of 81.

Simplifying Radicals

Square Roots

In order for a square root to be simplified, there can be no perfect square factors left in the radicand (under the square root.)

$\sqrt{75}$ $\qquad$ $\sqrt{48x^2}$ $\qquad$ $\pm\sqrt{245x^3y^6}$

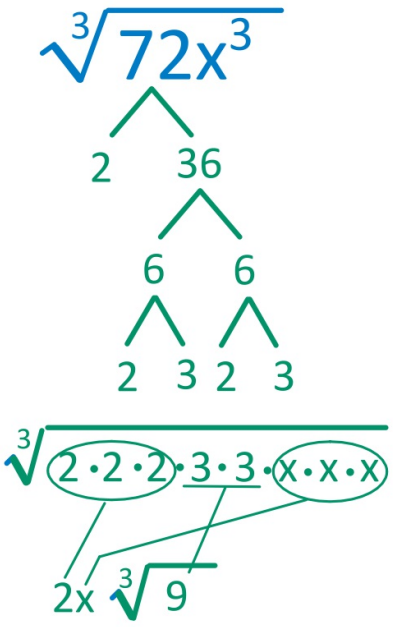


5 is part of a group of 2, so it comes out of the radical,
3 is not part of a group of 2, so it stays under.

Cube Roots

In order for a cube root to be simplified, there can be no perfect cube factors left in the radicand (under the square root.)

$\sqrt[3]{72x^3}$ $\qquad$ $\sqrt[3]{-128a^6b^5}$



2's and x's are part of a group of 3, so they come out of the radical. 3 is not part of a group of 3 so the 3 3 stays in.

Other Roots

Use the same methods as with the square roots and cube roots.

4th roots - look for groups of 4

5th roots - look for groups of 5

and so on...

$\sqrt[4]{243x^4y^5}$ $5\sqrt[5]{128a^3b^7}$

Rational Exponents

$8^{\frac{1}{3}} = \sqrt[3]{8^1}$ Simplify $= 2$

$c^{\frac{2}{5}}$ $(x^2)^{\frac{4}{3}}$